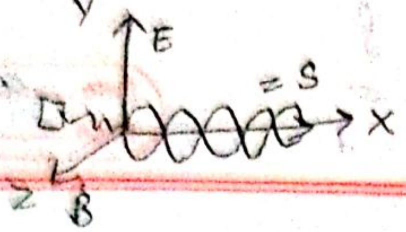
Significance of $\int J \cdot E \, dV$:-

When a charge of amount q_i is placed in an electromagnetic field then a Lorentz Force always act on the particle, which can be expressed as:

$$F_i = q_i[E_i + V_i \times B_i] \quad \text{--- (8)}$$

we know that the magnetic force act on the particle does not change its K.E. hence the work done by the magnetic force is equal



to zero. It means that the total work done by the electrostatic force and from the definition of work done, we know that

$$dW = F_i \cdot dl_i \quad \text{——— (9)}$$

where F_i is the Lorentz Force and dl_i is the displacement and we also know that

$$v_i = \frac{dl_i}{dt}$$

$$dl_i = v_i dt \quad \text{——— (10)}$$

Now from eqn (8), (9), (10), we get

$$dW = q_i [E_i + v_i \times B_i] \times v_i dt$$

$$dW = q_i E_i v_i dt + 0$$

$$\frac{dW}{dt} = q_i E_i v_i \quad \text{——— (11)}$$

This is the time rate of change of work done and can be expressed

volume = ρ
area = σ
length = λ

is the total energy of the charged particle.

The integral $\int J E \, dV$ can be expressed by using relations

$$J = \rho v$$

where ρ is the charge density, J is current density and v is the velocity of charged particle and we also know that

$$\rho = \frac{dq}{dV}$$

then above eq^n converts in the form of

$$\int J E \, dV = \int v E \frac{dq}{dV} \, dV$$

$$\boxed{\int J E \, dV = \int v E \frac{dq}{dV}}$$

$$\int J E \, dV = v E q \quad \text{——— (12)}$$

Now from eq^n (11), (12), we get

$$\int J E \, dV = v E q = \frac{dw}{dt} \quad \text{——— (13)}$$

Significance of $\int \frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\mu_0 H^2\right) dV$:-

The electrostatic energy in volume V can be expressed as

$$U_E = \int \frac{1}{2}\epsilon_0 E^2 \, dV$$

Similarly, magnetostatic energy in volume V can be expressed as :-

$$U_M = \int \frac{1}{2}\mu_0 H^2 \, dV$$

Then, the electromagnetic energy can be expressed as :-

$$U_{EM} = U_E + U_M$$

$$U_{EM} = \int \left(\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\mu_0 H^2\right) dV$$

On differentiating w.r.t 't', we get

$$\boxed{\frac{\partial U_{EM}}{\partial t} = \int \frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\mu_0 H^2\right) dV} \quad (14)$$

Now, putting the value of eqn (14) and (13), in eqn (7), we get

$$\frac{dw}{dt} + \frac{\partial U_{EM}}{\partial t} = -\int (E \times H) \, ds$$

or $$\boxed{\frac{dW}{dt} = -\frac{\partial U_{EM}}{\partial t} - \int (E \times H) \cdot ds}$$

This is the conservation law of energy, which shows that rate of change of electromagnetic energy decreases in electromagnetic is equal to the rate of change of work done by the charge particle which is also known as total energy and rest energy is emitted in the form of electromagnetic energy by the surface S.

Also $(E \times H) = S$ is known as poynting vector

whose magnitude is equal to the product of electric field intensity and magnetic field intensity, then the above eqn can be written as :-

$$\boxed{\frac{dW}{dt} = -\frac{dU_{EM}}{\partial t} - \int S \, ds}$$

The poynting vector S has its own significance i.e; $\vec{S} = (\vec{E} \times H)$

or $\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$

i.e; the electric field and magnetic field is perpendicular to each other.
Hence, the poynting vector is also perpendicular to the electric & magnetic field and can be shown as

$$\vec{S} = |E||H| \sin\theta$$

magnitude only (neglecting angle) → $\boxed{|S| = |E||H|}$